Group theoretical approach to the equivalence principle for uniformly accelerated frames

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Abstract. *This paper discusses the equivalence principle (classical and quantum) in its simplest form (i.e., for uniformly accelerated frames) from the viewpoint of the cohomology of groups.*

1. INTRODUCTION

Many discussions exist concerning the relevance of non-inertial frames for the description of physical phenomena. Classically such discussions are often couched in the language of various equivalence principles and these are sometimes elevated to postulates that permeate quantum mechanics and quantum field theory. In an attempt to make more precise such imprecise notions we wish to point out that a natural cohomological explanation exists for the absence of a well-defined equivalence principle in a relativistic framework.

We examine first the dynamics of massive particles under the influence of a constant force in a Newtonian inertial frame. The existence of a global frame of

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reference in which the particle appears free is traced to the cohomological structure of the Galilei group (the invariance group of Newtonian mechanics) in one space and time dimensions allowing for non-trivial $U(1)$ extensions (the quantum group of the Schrödinger equation). A relativistic generalisation is obtained by adopting the (1, 1) Poincaré group (the isometry group of (1, 1) Minkowski space) as dynamical group for the classical mechanics and its non-trivial $U(1)$ extensions to describe the quantisation. The basic observation is that these extensions are characterised by an interaction parameter (that might for example be interpreted as the constant force experienced by an electric charge coupled to a uniform electric field in an inertial Minkowski frame) that can never be transformed to zero by a change of frame. Unlike the non-relativistic case the relevant group cohomology is only one-dimensional rather than two and no freedom exists to «remove» the interaction. Thus it is only in a Galilean (or «non-relativistic») setting that one can properly compensate the effects of a constant Newtonian force in an inertial frame by transforming the Schrödinger Hamiltonian to a frame of reference in uniform Newtonian acceleration. By contrast one may never find in Minkowski space a coordinate patch in which a coupled Klein-Gordon equation can be interpreted as being non-interacting.

2. DYNAMICS OF A GALILEAN PARTICLE IN THE PRESENCE OF A CONSTANT FORCE

The classical mechanics of a non-relativistic particle of mass m in the presence of a constant force may be described by a variational principle based on the so-called *Poincaré-Cartan (PC) form*, which may be written as

$$\Theta_{PC} = -x\,\mathrm{d}p - \frac{p^2}{2m}\,\mathrm{d}t + \frac{F}{2}(x\,\mathrm{d}t - t\,\mathrm{d}x). \tag{2.1}$$

Although (2.1) does not coincide with the more familiar expression $\Theta_{PC} = p\,\mathrm{d}x - H\,\mathrm{d}t$ where $H = \frac{p^2}{2m} - Fx$, the difference is an (irrelevant) exact form. Indeed, with either Θ_{PC}, the solutions to the equations of the motion are given by cross sections of the bundle $\mathbb{R} \times T^*(M) \to \mathbb{R}$, – where $\mathbb{R}$ is parameterised by the absolute Newtonian time t and $M(=\mathbb{R})$ by x – which satisfy $i_X\,\mathrm{d}\Theta|_{s^*} = 0\ \forall X$, where $X = X^t \frac{\partial}{\partial t} + X^x \frac{\partial}{\partial x} + X^p \frac{\partial}{\partial p}$ is a general vector field on $\mathbb{R} \times T^*(M)$ and $s^* = (x(t), p(t))$.

A geometric description of the corresponding quantum system may start from the consideration of the *generalised quantisation form* on $(R \times T^*(M)) \times S^1 = \widetilde{G}$,

$$\Theta = -x\,dp - \frac{p^2}{2m}\,dt + \frac{F}{2}(x\,dt - t\,dx) + \hbar\,\frac{d\zeta}{i\zeta} \tag{2.2}$$

where ζ globally parameterises $S^1 = U(1)$. The pair $(\widetilde{G}, \Theta)$ is called [1] a *generalised quantum manifold*; it differs from the Kostant-Souriau quantum manifold [2] in that, because $\widetilde{G}$ contains the time, $(\widetilde{G}, \Theta)$ is not a contact manifold. A theory based on (2.1) [(2.2)] will be said to be associated with the classical [quantum] particle of mass m under the influence of a constant force F.

Both the classical and quantum description based on (2.1), (2.2) may be unified [1] if $\widetilde{G}$ is taken to be a *group* manifold. In the present case of the constant force, this manifold turns out to be [3] the central $U(1)$-extension of the Galilei group $G^{(1,1)}$ in one space and time dimensions given by the following group law

$([t'', x'', V''; \zeta''] = [t', x', V'; \zeta']^* [t, x, V; \zeta])$:

$$t'' = t' + t$$

$$x'' = x' + x + V't \tag{2.3a}$$

$$V'' = V' + V$$

$$\zeta'' = \zeta'\zeta \exp \frac{i}{\hbar}\left\{m\left[x'V + t\left(V'V + \frac{1}{2}V'^2\right)\right] + \frac{F}{2}\left[(t'x - tx') + V't't\right]\right\}. \tag{2.3b}$$

The group defined by (2.3) is labelled $\widetilde{G}_{(m,F)}$, where the ~ refers to the fact that it is a $U(1)$ extension and (m, F) determine an element (class) in the two dimensional $H^2(G^{(1,1)}, U(1))$ cohomology vector space to which the non-trivial 2--cocycle appearing in (2,3b) belongs. It should be noted that the cocycle has the dimensions of an action; this, together with (2.5) below, fixes for the group parameters the customary dimensions. We shall put $\hbar = 1$ henceforth.

From (2.3) we may compute the left-invariant vector fields (LIVF) and the (left) $\widetilde{\mathscr{G}}_{(m,F)}$ algebra. We obtain

$$\widetilde{X}^L_{(t)} = \frac{\partial}{\partial t} + V\frac{\partial}{\partial x} + \left(\frac{1}{2}mV^2 - \frac{F}{2}(x - Vt)\right)\Xi$$

$$\widetilde{X}^L_{(x)} = \frac{\partial}{\partial x} + \frac{F}{2}t\,\Xi \tag{2.4}$$

$$\widetilde{X}^L_{(V)} = \frac{\partial}{\partial V} + mxV\Xi; \qquad \widetilde{X}^L_{(\zeta)} \equiv \Xi = i\zeta\frac{\partial}{\partial \zeta}$$

and

$$[\widetilde{X}^L_{(V)}, \widetilde{X}^L_{(t)}] = \widetilde{X}^L_{(x)}, [\widetilde{X}^L_{(x)}, \widetilde{X}^L_{(V)}] = m\Xi,$$
(2.5)
$$[\widetilde{X}^L_{(t)}, \widetilde{X}^L_{(x)}] = F\,\Xi, [\Xi, \text{any } \widetilde{X}^L] = 0$$

where Ξ is the central $U(1)$ subgroup generator. Once the group law is stated, the generalised quantisation form is obtained from the (vertical) component of the canonical LI form on $\widetilde{G}_{(m,F)}$. The one-form dual to the (fundamental) vertical vector field Ξ of the principal bundle $\widetilde{G}_{(m,F)}(U(1), G^{(1,1)})$ is determined by the conditions

(2.6) $$\Theta(\Xi) = 1, \ \Theta(\text{any other } \widetilde{X}^L) = 0.$$

Indeed, one easily obtains (2.2) from (2.6) and (2.4) ($p \equiv mV$), and then it is possible to *define* $\Theta_{PC} = \Theta - d\zeta/i\zeta$ *directly from the group manifold* (and, of course, obtain (2.1)). Because the addition of a 2-coboundary does not modify the extension [4], there is a certain amount or arbitrariness in the explicit form of the non-trivial 2-cocycle appearing in (2.3b) which accounts for the class of equivalent Θ's (for a given pair m, F) which differ by an exact form (*).

The group manifold approach to quantisation now leads to the wave-equation by imposing on a general function $\Psi : \widetilde{G} \to \mathbb{C}$ on the group manifold the conditions of being a) $U(1)$-equivariant ($\Xi, \Psi = i\Psi$) and b) annihilated by the generators of the *maximal polarisation subalgebra* Π which includes the LI vector fields generating the characteristic module $\mathscr{C}_\Theta$ of Θ ($\mathscr{C}_\Theta = \{\widetilde{X} \mid i_{\widetilde{X}}\Theta = 0 = i_{\widetilde{X}}d\Theta\}$) and is defined as the maximal horizontal ($\Theta(\widetilde{X}^L) = 0 \ \ \forall \widetilde{X}^L \in \Pi \subset \widetilde{\mathscr{G}}_{(m,F)}$) subalgebra of $\widetilde{\mathscr{G}}_{(m,F)}$. The final result, which is given in terms of the momentum representation is, of course, the familiar equation

(2.7) $$\left(i\,\frac{\partial}{\partial t} - \frac{p^2}{2m} + F\,i\,\frac{\partial}{\partial p}\right)\phi(t,p) = 0.$$

We remark that (2.1), (2.2) and (2.7) are obtained by having the group $\widetilde{G}_{(m,F)}$ as the only starting point. (It is also possible to have a definition of the classical limit in the Hamilton-Jacobi version starting from $\overline{G}_{(m,F)}$, the group which is obtained when $(U(1), \cdot)$ is replaced by $(\mathbb{R}, +)$, but this will not concern us here).

(*) *Note added in proof*: For a review, see V. Aldaya and J.A. de Azcárraga, *Group Foundations of Quantum and Classical Dynamics: towards a globalization and classification of some of their structures*, to appear in Fortschr. der Physik (1987).

3. COHOMOLOGY, THE CONSTANT FORCE AND THE EQUIVALENCE PRINCIPLE IN GALILEAN MECHANICS

Expression (2.3b) provides us with a cohomological definition of the mass m and of the constant force F (or constant acceleration $a = F/m$); they are the two real parameters which characterise the second cohomology group (2-dimensional vector space) $H^2(G^{(1,1)}, U(1))$ which in turn characterises the central extensions $\widetilde{G}_{(m,F)}$ of $G^{(1,1)}$ by $U(1)$. The mathematical role of the mass parameter has been known since the work of Bargmann [4]; to analyse that of F, we first consider a simple and unseemingly related problem.

Let us look for the most general group parameter transformation which leaves the Galilei group $G^{(1,1)}$ group law invariant and thus induces the identity in its Lie algebra, $G^{(1,1)}$. The general transformation (where $a, b, \ldots, k$ are dimensionless constants) is

$$
\begin{aligned}
t &\equiv a\hat{t} + b\frac{m}{F}\hat{V} \\
x &\equiv c\hat{x} + e\hat{V}\hat{t} + f\frac{m}{F}\hat{V}^2 + g\frac{F}{m}\hat{t}^2 \qquad (3.1)\\
V &= j\hat{V} + k\frac{F}{m}\hat{t}
\end{aligned}
$$

and the condition that the new (ˆ) variables fulfill again (2.3a) gives

$$
\begin{aligned}
t &= a\hat{t} + b\frac{m}{F}\hat{V} \\
x &= (aj - e)\hat{x} + e\hat{V}\hat{t} + \frac{1}{2}bj\frac{m}{F}\hat{V}^2 + \frac{1}{2}\frac{ae}{b}\frac{F}{m}\hat{t}^2 \qquad (3.2)\\
V &= j\hat{V} + \frac{e}{b}\frac{F}{m}\hat{t}
\end{aligned}
$$

with $(aj - e) \neq 0$, and $a \neq \frac{e}{b}$ if $b = j$, the unit transformation being given by $a = j = 1$ and all others being zero. Because the new variables preserve the group law, the 2-cocycle structure of (2.3b) is retained in the new variables. We may then compute the $\widetilde{G}_{(m,F)}$ LIVF in terms of the new variables as well as the $\widetilde{\mathscr{G}}_{(m,F)}$ Lie brackets, with the result (cf. 2.5))

$$
\begin{aligned}
&[\widetilde{X}^L_{(\hat{V})}, \widetilde{X}^L_{(\hat{t})}] = \widetilde{X}^L_{(\hat{x})} \qquad [\widetilde{X}^L_{(\hat{x})}, \widetilde{X}^L_{(\hat{V})}] = m(ja - e)(j - b)\,\Xi \\
&[\widetilde{X}^L_{(\hat{t})}, \widetilde{X}^L_{(\hat{x})}] = F(ja - e)\left(a - \frac{e}{b}\right)\Xi \qquad [\Xi, \text{any } \widetilde{X}^L] = 0.
\end{aligned}
\qquad (3.3)
$$

A particular case of the above [(3.2)] is obtained for $a = e/b = 1$, $e = b = 0$, $j = 1$. Then (3.3) gives (2.5) with the third commutator replaced by

$$[\tilde{X}^L_{(\hat{t})}, \tilde{X}^L_{(\hat{x})}] = 0. \tag{3.4}$$

This transformation corresponds to choosing a basis in the cohomology vector space $H^2(G^{(1,1)}, U(1))$ for which the vector (m, F) becomes $(m, 0)$.

The interesting physical consequence is that this transformation, which eliminates the force, *and which exists because of the two-dimensional nature of the group cohomology*, provides a group theoretical interpretation of a Galilean («non-relativistic») equivalence principle for an observer moving with constant acceleration (with respect to a Newtonian inertial frame). Indeed, on the $G^{(1,1)}$ manifold the transformation which eliminates F is none other than

$$t = \hat{t}, \quad x = \hat{x} + \frac{1}{2}\frac{F}{m}\hat{t}^2 \tag{3.5a}$$

$$V = \hat{V} + \frac{F}{m}\hat{t} \tag{3.5b}$$

where the variables with   define an accelerated coordinate system. The change (3.5a) brings – up to an exact one-form – (2.1) to the «free» form

$$\Theta_{PC} = -\hat{x}\,\mathrm{d}\hat{p} - \frac{\hat{p}^2}{2m}\,\mathrm{d}\hat{t} \tag{3.6}$$

and (3.5) also transforms (2.2) into

$$\Theta = -\hat{x}\,\mathrm{d}\hat{p} - \frac{\hat{p}^2}{2m}\,\mathrm{d}\hat{t} + \frac{\mathrm{d}\hat{\zeta}}{i\hat{\zeta}} \tag{3.7}$$

where $\hat{\zeta}$ is the new $U(1)$ parameter whose relation with ζ may be obtained by substituting (3.5) into (2.3b) ((3.7) is nevertheless obvious since Θ is the vertical [(2.6)] one-form for the algebra (2.5) with $F = 0$ [(3.4)]).

To conclude this section we may establish the connection among the above and the approach to the quantum equivalence principle in Galilean wave mechanics. There, it obviously means the existence of a transformation going from the Fourier transform of (2.7),

$$\left(i\frac{\partial}{\partial t} + \frac{1}{2m}\frac{\partial^2}{\partial x^2} + Fx\right)\phi(t, x) = 0 \tag{3.8}$$

to

$$\left(i\frac{\partial}{\partial t} + \frac{1}{2m}\frac{\partial^2}{\partial x^2}\right)\hat{\phi}(\hat{t}, \hat{x}) = 0 \tag{3.9}$$

exhibiting that a constant force F in an inertial reference system can be eliminated by transforming the equation to a uniformly accelerated frame. This is certainly achieved by the space-time change of variables (3.5a) *provided* that we allow for a phase transformation, i.e.

$$\hat{\phi}(\hat{t}, \hat{x}) = \exp i\, f(t, x)\, \phi(t, x) \tag{3.10}$$

which turns out to be (1)

$$f(t, x) = -Ftx + \frac{F^2 t^3}{6m}\,. \tag{3.11}$$

The cohomological origin of this phase is now clear: although the non-inertial transformation (3.5a) is not a symmetry of the Schrödinger equation, it still allows us to pass from (3.8) to (3.9) because as already mentioned the transformation (3.5) of the group variables induces a change of basis in the $H^2(G^{(1,1)} U(1))$ vector space so that (m, F) is now represented by $(m, 0)$. (Although also of cohomological origin, this phase should not be confused with that which is required when going to inertial systems S' moving with velocity V with respect to S, $\phi'(t', X') = \exp i\left(mVx + \frac{1}{2} mV^2 t\right)\phi(t, x)$, which is associated with the Galilean transformation generated by the boosts subgroup, and not with a constant as F). The origin of (3.10) puts in a new perspective the fact that one may find «accelerated» solutions $\hat{\phi}(t, x)$ of the «free» Schrödinger equation (3.9) (see e.g. [5, 6]). To interpret a solution to the Schrödinger equation one should declare in advance what frame of reference is being used to represented the solution. If the frame is *declared* inertial (in the Newtonian sense), then the presence of a constant real force can be compensated by the space-time non-inertial effects implied by (3.5a), (3.10) leaving a free particle equation in the new frame.

4. THE RELATIVISTIC CASE

To discuss the relativistic motion of a particle and its quantisation we shall seek a group which is an extension of the Poincaré group and which gives (2.3) in the non-relativistic (contraction $c \to \infty$) limit. Although the ordinary Poincaré group associated with the (1,3) Minkowski metric of space-time has trivial cohomology, the (1,1) Poincaré group has non-trivial $H^2(P^{(1,1)}, U(1))$ cohomology and may be extended non-trivially (i.e., not by direct product) by $U(1)$. The interesting

(1) In momentum space, one similarly has

$$\hat{\phi}(\hat{t}, \hat{p}) = \exp i\left(\frac{F}{2m} pt^2 - \frac{F^2}{3m} t^3\right) \phi(t, p).$$

fact is that in this case $H^2(P^{(1,1)}, U(1))$ is a one-dimensional space, the elements of which we shall characterise by F. It is nevertheless possible to introduce the mass m through a *2-coboundary* belonging to the class of 2-coboundaries which become non-trivial 2-cocycles in the non-relativistic limit. (In this sense, these coboundaries are «less trivial» than those which remain 2-coboundaries in that limit and may be said to belong to a pseudo-cohomology class which, in the $c \to \infty$ limit, becomes a cohomology one parametrised by m).

We propose the following group, labelled $\widetilde{P}_{(F)m}$, to describe the relativistic particle moving under a constant acceleration of magnitude $a = F/m$ along the real line

$$x_0'' = x_0' + x_0 \,\mathrm{ch}\,\chi' + x\,\mathrm{sh}\,\chi'$$
$$(4.1a) \qquad x'' = x'' + x_0' \mathrm{sh}\,\chi + x\,\mathrm{ch}\,\chi'$$
$$\chi'' = \chi' + \chi$$

$$\zeta'' = \zeta'\zeta \exp i\left\{\frac{F}{2}\,[(x_0'x - x_0x')\,\mathrm{ch}\,\chi' + (x_0'x_0 - x'x)\,\mathrm{sh}\,\chi'] + \right.$$
$$(4.1b) \qquad \left. + mc[(x''\,\mathrm{sh}\,\chi'' - x_0'') - (x'\,\mathrm{sh}\,\chi' - x_0') - (x\,\mathrm{sh}\,\chi - x_0)]\right\}$$

where $x^0 = ct$, χ parameterises the boosts (other useful parameterisations are $\mathrm{ch}\chi = (1 - V^2/c^2)^{-1/2}$, $\alpha \equiv \mathrm{sh}\,\chi/2$, $p_0 = mc(1 + 2\alpha^2) \equiv mc\,\mathrm{sh}\,\chi$, $p \equiv 2mc\alpha(1 + \alpha^2)^{1/2} \equiv mc\,\mathrm{ch}\,\chi$). One also notices the 2-coboundary structure of the exponent in m in (4.1b). It may be easily checked that $\lim_{c\to\infty} \widetilde{P}_{(F)m}$ [(4.1)] is $\widetilde{G}_{(m,F)}$ [(2.3)] as it should. We use the label $(F)m$ to emphasise that, since m is associated with a 2-coboundary, only F characterises the non-trivial cocycle of $H^2(P^{(1,1)}, U(1))$. In the contraction ($c \to \infty$) limit, (4.1) becomes its non-relativistic counterpart (2.3), the mass 2-coboundary becoming a *non-trivial* 2-cocycle.

Following the same scheme of Sec. 3 we find the Lie algebra commutators

$$[\widetilde{X}^L_{(\alpha)}, \widetilde{X}^L_{(x^0)}] = 2\widetilde{X}^L_{(x)} \quad [\widetilde{X}^L_{(x)}, \widetilde{X}^L_{(\alpha)}] = -2(\widetilde{X}^L_{(x^0)} - mc\Xi)$$
$$(4.2)$$
$$[\widetilde{X}^L_{(x^0)}, \widetilde{X}^L_{(x)}] = \frac{F}{c}\,\Xi \quad [\Xi, \text{any } \widetilde{X}^L] = 0$$

(again, $\lim_{c\to\infty} \widetilde{\mathscr{P}}_{(F)m}$ [(4.2)] $= \widetilde{\mathscr{G}}_{(m,F)}$ [(2.5)]). The form Θ dual to Ξ gives the generalised quantisation form,

$$(4.3) \qquad \Theta = -x\,\mathrm{d}p - (p^0 - mc)\,\mathrm{d}x^0 + \frac{F}{2c}(x\,\mathrm{d}x^0 - x^0\,\mathrm{d}x) + \frac{\mathrm{d}\zeta}{i\zeta}$$

which, because the appearance (of the otherwise exact) $mc\,dx^0$ term, has (2.2) as the non-relativistic limit. To see that (4.3) describes a particle submitted to a constant force it is sufficient to evaluate the classical trajectories from the associated Poincaré-Cartan form or, equivalently, to integrate the non $U(1)$--components of the characteristic module (Sec. 2) which turns out to be generated by the vector field

$$C = \tilde{X}^L_{(x^0)} + \frac{F}{2mc^2}\tilde{X}^L_{(\alpha)} \tag{4.4}$$

where (2)

$$\tilde{X}^L_{(x^0)} = \frac{p_0}{mc}\frac{\partial}{\partial x^0} + \frac{p}{mc}\frac{\partial}{\partial x} + \left\{\frac{F}{2mc^2}(x_0 p - x p_0) + \frac{p_0 - mc}{mc}p_0\right\}\Xi \tag{4.5a}$$

$$\tilde{X}^L_{(\alpha)} = 2p_0\frac{\partial}{\partial p} + 2p_0 x\,\Xi. \tag{4.5b}$$

The result is, for a suitable pair of initial conditions,

$$x^0 = \frac{mc^2}{F}\,\mathrm{sh}\left(\frac{F}{mc}\tau\right) \qquad x = \frac{mc^2}{F}\,\mathrm{ch}\left(\frac{F}{mc}\tau\right) \tag{4.6}$$

i.e., the branch of a hyperbola corresponding to a motion with acceleration $a^\mu = \dfrac{d^2x^\mu}{d\tau^2}$, $a^2 = (F/m)^2$.

We may now try to follow the same line of reasoning of section 3 to eliminate F. But we immediately notice that the local transformation to the accelerated ($\hat{\ }$) frame,

$$\begin{aligned} x^0 &= \left(\frac{mc^2}{F} + \hat{x}\right)\mathrm{sh}\left(\frac{\hat{x}^0 F}{mc^2}\right) \\ x &= \left(\frac{mc^2}{F} + \hat{x}\right)\mathrm{ch}\left(\frac{\hat{x}^0 F}{mc^2}\right) \end{aligned} \tag{4.7}$$

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(2) We give here, for completeness, the remaining non-trivial generator

$$X^L_{(x)} = \frac{p}{mc}\frac{\partial}{\partial x^0} + \frac{p_0}{mc}\frac{\partial}{\partial x} + \left\{\frac{a}{2c^2}[p_0 x_0 - px] + \frac{p_0 - mc}{mc}p\right\}\Xi \tag{4.5c}$$

One may check that, in the limit $c \to \infty$, (4.5) leads to (2.4) and C to $X^L_{(t)} + \dfrac{F}{m}X^L_{(V)}$, which generates the characteristic module of (2.2).

whose $c \to \infty$ limit precisely gives the space-time transformations of (3.5a), cannot be completed with a transformation in the boost parameter χ to obtain a transformation preserving the (1,1) Poincaré group law (which can be read from (4.1a)). In an analogous fashion, there is no transformation in the relativistic (Poincaré) case which takes the canonical one-form (4.3) to a Θ with $F = 0$.

Again, this fact is of cohomological character and may be proven with generality from the algebra $\widetilde{\mathscr{P}}_{(F)m}$. Because the space H^2 of cohomology is of dimension one, it is not possible to put $F = 0$ in the present relativistic case *without changing the group* and accordingly the dynamical system. Indeed, for $F = 0$ one has a group with a direct product structure (and hence not isomorphic to $\widetilde{\mathscr{P}}_{(F)m}$). That this cannot (obviously) be the case may be seen by tyring to get the r.h.s. of $[\widetilde{X}^L_{(x^0)}, \widetilde{X}^L_{(x)}]$ in (4.2) equal to zero by redefining the basis of the algebra: this leads immediately to a contradiction, in contrast with the Galilei case where the redefinitions $\widetilde{X}^L_{(\hat{t})} = \widetilde{X}^L_{(t)} + \frac{F}{m}\widetilde{X}^L_{(V)}$, $\widetilde{X}^L_{(\hat{x})} = \widetilde{X}^L_{(x)}$, $\widetilde{X}^L_{(\hat{V})} = \widetilde{X}^L_{(V)}$ achieve the result (3). Thus we may conclude that the different role played by the constant accelerated frames is of cohomological origin.

5. CONCLUSIONS

As is known, the Newtonian and Schrödinger equations are not tensorial equations on a space-time manifold (their evolution is with respect to a preferred absolute Galilean time). In this paper we have put forward a group approach to the problem of discussing their behaviour under the transformation to a constant accelerated frame of reference, based on adopting the extended Galilei group $\widetilde{G}_{(m,F)}$ as the dynamical group characterising the system. We have shown that the possibility of writing the classical and quantum equations in a «free» form, can be attributed to the group cohomological structure of the Galilei group in one time and space dimensions, the space direction being singled out by the acceleration. In relativistic physics, by contrast, classical and quantum mechanics may be formulated tensorially on space-time with prefereed frames existing only in terms of Killing vector fields. Because of the different cohomological structure of the Poincaré group in two dimensions, a conclusion similar to that of the Galilean

(3) Of course, the fact that the Θ of (4.3) cannot be brought to the form $\Theta(F = 0) = -x\,dp - (p^0 - mc)dx^0 + d\zeta/i\zeta$ does not preclude that we may locally bring C in (4.4) to $\widetilde{X}^L_{(x^0)}$ [(4.5a)], which is the vector field which generates the characteristic module of $\Theta(F = 0)$. (In fact, any regular vector field may be written in a local chart in which its integral curves are constant coordinate lines). However, the generalised quantisation form depends on the whole algebra, and F cannot be set equal to zero in $\widetilde{\mathscr{P}}_{(F)m}$ without changing its structure.

situation cannot be drawn, reinforcing the idea that there is no analogous «equivalence principle» for the description of interacting relativistic wave equations.

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